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# Derivatives

## Base Correlations

### *Overcoming the Limitations of Tranche Implied Correlations*

**Global**

#### **Tranche Implied Correlations Have Limitations...**

Though changes in tranche implied correlations are a good indicator of demand for the tranches, the implied correlations themselves are limited by the underlying model and the existence of a skew. These limitations include:

- Difficulty in pricing non-standardized tranches;
- Existence of multiple implied correlations for the mezzanine tranche.

#### **...That Are Addressed by Base Correlations**

These limitations can be overcome by base correlations, which are defined as implied correlations of first-loss tranches where the attachment point is zero and the detachment point is the detachment point of the various standardized tranches.

Base correlations are relatively easy to compute. They can be extracted from market spreads of the standardized tranches via bootstrapping techniques using the same model that is used to derive tranche implied correlations.

Base correlations are unique and provide an intuitive method to price non-standardized tranches (say 5-8%) of the underlying CDX or iTraxx portfolio.

#### **Base Correlation Deltas**

The tranche delta or leverage of a tranche depends on whether we use tranche implied or base correlations. Under the base correlation framework the leverage tends to be lower especially for the mezzanine tranche. We discuss how deltas are calculated under both frameworks.

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*Tranche implied correlations are limited by the underlying model and correlation skew*

**From Tranche Implied Correlations...**

In an earlier report<sup>1</sup>, we had highlighted the limitations of tranche implied correlations. A key problem is the pricing of non-standardized or bespoke tranches. The correlation skew in the standardized market makes it difficult to price off-market tranches such as, for example, a 5-8% tranche of the CDX.NA.IG3 index. In addition, pricing of tranches based on portfolios different from the CDX.NA.IG3 also requires an appropriate estimate of the default correlation. Another limitation of the tranche implied correlation is the existence of multiple (or non-unique) solutions for implied correlations for mezzanine tranches.

These limitations arise due to the use of the standard Gaussian Copula model and the existence of a correlation skew in the standardized tranche market. The market continues to debate the use of an appropriate measure for default correlation. We believe this debate is unlikely to be completely resolved in the near future and urge tranche investors to look at correlation trends rather than absolute levels of correlation. Irrespective of the measurement or model used, the correlations trends serve as a good indicator of demand in the tranche market.

**...to Base Correlations**

*Base correlations correspond to first-loss tranches with standardized detachment points*

One of the methods that is commonly used in the market to address the above issues is the use of base correlations. Base correlations are defined as the implied correlations of the **first-loss** tranches where the attachment point of each tranche is zero and the detachment point is the detachment point of each of the standardized tranches. Table 1 highlights the tranche implied correlations for the standardized tranches and the corresponding first-loss implied base correlations.

**Table 1: Base Correlations & Tranche Implied Correlations for Standardized Tranches**

Std. Tranche	North America (CDX.NA.IG3)					Europe (iTraxx Europe Series 2)							
	Bid	Ask	Mid	Tranche Correlation	Base Tranches	Base Correlation	Std. Tranche	Bid	Ask	Mid	Tranche Correlation	Base Tranches	Base Correlation
0-3%	34.25%	35.25%	34.75	19.1%	0-3%	19.1%	0-3%	23.25%	24.5%	23.875%	19.0%	0-3%	19.0%
3-7%	221	227	224	4.1%	0-7%	29.6%	3-6%	132	138	135	5.2%	0-6%	28.8%
7-10%	87	91	89	18.1%	0-10%	34.2%	6-9%	43	47	45	13.4%	0-9%	36.4%
10-15%	28	33	30.5	20.1%	0-15%	43.3%	9-12%	28	33	30.5	21.9%	0-12%	41.7%
15-30%	9.25	10.25	9.75	29.7%	0-30%	66.5%	12-22%	14	16	15	30.9%	0-22%	54.8%

Source: Merrill Lynch; As of 9<sup>th</sup> Nov 2004; All 0-3% tranches quoted on Upfront + 500bps running basis.

*Overcome the key limitations of tranche implied correlations*

Base correlations overcome the key limitations of tranche implied correlations:

- Base correlations can be used to price tranches with customized attachment and detachment points of either standardized or bespoke underlying portfolios.
- Base correlations are unique across the capital structure.

Base correlations can be computed with relative ease using the same model that is used to derive tranche implied correlations. We also highlight in the following sections how base correlations can be used to price non-standard tranches of the underlying CDX or iTraxx portfolios.

<sup>1</sup> For an in-depth discussion on the technicals affecting the skew see “Correlation Skew” by Kakodkar/Galiani et al, published 14<sup>th</sup> July 2004.

*Use bootstrapping method to derive base correlations from market spreads*

### ■ Deriving Base Correlations of Standardized Tranches

Base correlations can be derived directly from the market spreads of the standardized tranches using standard bootstrapping techniques. The base correlation of the first-loss 0-3% tranche is obviously the same as its tranche implied correlation. However, the 0-7% tranche can be derived in the following standard manner:

- Price the 0-7% tranche as a combination of the 0-3% and the 3-7% assuming both tranches have a premium equal to that of the 3-7% market premium.
- Price of 3-7% tranche using the 3-7% market premium is zero.
- Price of 0-7% tranche is therefore equal to the price of 0-3% with the same premium as the 3-7% market premium, using the 0-3% base correlation (also the tranche implied correlation in this case). The price will be positive since the 3-7% premium is smaller than the 0-3% market premium.
- Use this price for the 0-7% (after adjusting for the notionals) and the standard Gaussian Copula model to imply the base correlation of the 0-7% tranche.

In the same manner, we can imply the 0-10% base correlation by pricing the 0-10% tranche as a combination of the 0-7% tranche and the 7-10% assuming both tranches have a premium equal to that of the 7-10% market premium. This procedure is then repeated to derive 0-15% and 0-30% base correlations.

In order to illustrate the implementation of the above procedure, let us compute the implied base correlation for the 0-7% tranche. As explained above, we start by computing the price of the 0-3% tranche using the 3-7% tranche premium with the 0-3% base correlation. Given the levels in Table 1, this value, expressed in terms of the 0-3% notional, equals 45.1%.

*Scaling each intermediate step by the corresponding notional is crucial*

In the next step we compute the implied 0-7% correlation which matches the value of the 0-7% tranche using the 3-7% premium. With this regard, we first need to re-state the 0-7% tranche value by **scaling** the above number (the 0-3% mark-to-market using the 3-7% premium) by the appropriate notional. This can be easily done by multiplying the value of the 0-3% tranche (with the 3-7% premium) by the ratio of the width of the 0-3% tranche (3%) and the width of the 0-7% tranche (7%). The required calculations are in shown in Table 2.

**Table 2: Notional Adjusted Mark-to-Market**

0-7% Tranche Value (Notional Scaled)	
0-3% tranche value using the 3-7% premium	45.1%
Width Ratio	$3\% / 7\% = 0.43x$
0-7% tranche value using the 3-7% premium	19.39%
0-7% base correlation	29.6%

Source: Merrill Lynch

*0-7% Notional Scaled Value*

$$45.1\% \times 0.43 = 19.85\%$$

Once the notional-adjusted 0-7% mark-to-market has been computed, we can then apply the standard Gaussian Copula Model to find the correlation that matches the tranche value. Using a simple trial-and-error procedure in this example, we obtain an implied base correlation of 29.6% for the 0-7% tranche.

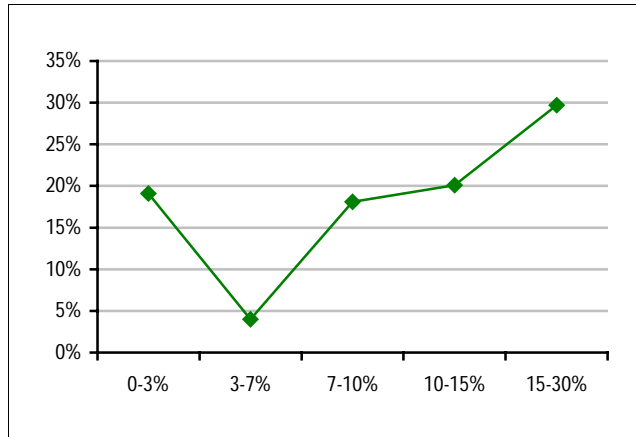
We can then recursively compute the implied base correlation for the remaining tranches. With regard to the 0-10% tranche, we first need to compute the value of the 0-7% tranche using the 7-10% premium and the 0-7% base correlation. This implies a 0-7% mark-to-market of 25.18%. Given the width ratio of 0.7x (0-7% width / 0-10% width) we can then compute the notional adjusted 0-10% mark-to-market which is equal to 17.6% (25.18% x 0.7). Given this value we can apply again the Gaussian Copula model and solve for the implied 0-10% base which, as reported in Table 1, equals 34.2%.

**■ Pricing Non-Standard Tranches of Standardized Indices**

*Correlation skew makes it difficult to price non-standard tranches of the index*

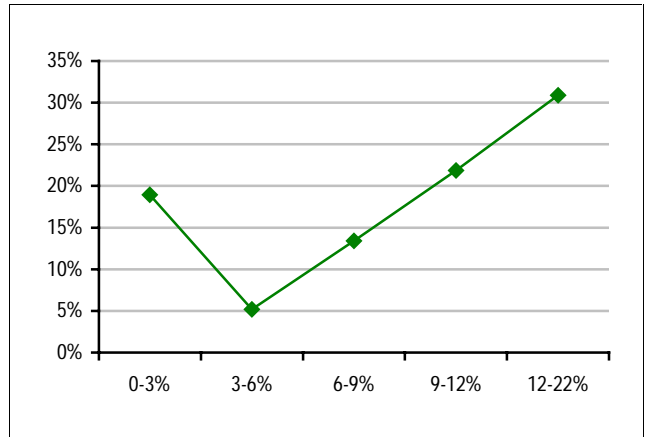
One of the key problems with tranche implied correlations is the pricing of non-standard tranches such as a 5-8% tranche of the CDX index. For the standardized tranche market in North America and Europe, we observe that tranche implied correlations are skewed in the form of a correlation “smile” (see Chart 1 and Chart 2). Due to this skew, it is not very clear what correlation input should be used to derive the price of a non-standard tranche.

**Chart 1: Tranche Implied Correlations (CDX.NA.IG)**



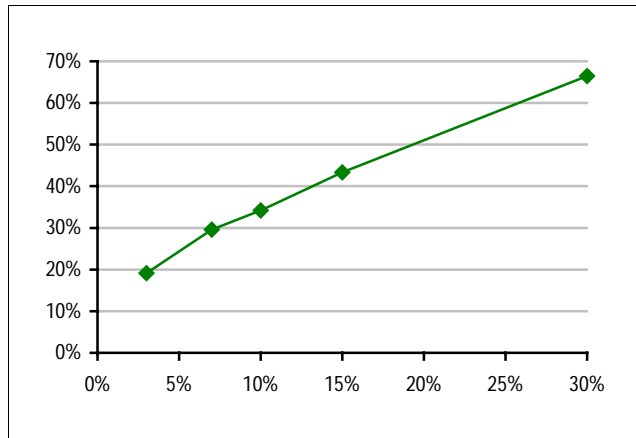
Source: Merrill Lynch

**Chart 2: Tranche Implied Correlations (iTraxx Europe)**



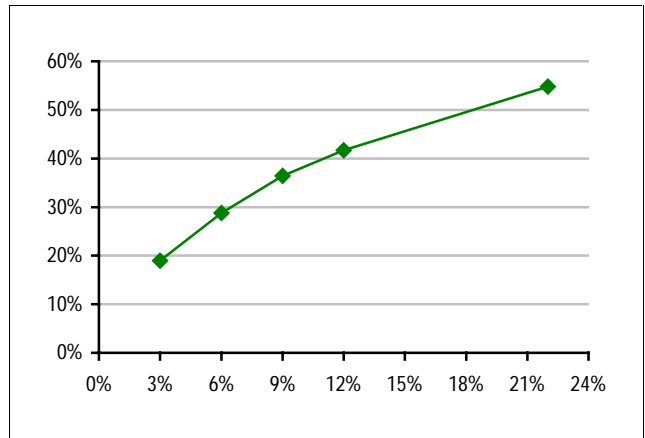
Source: Merrill Lynch

**Chart 3: Base Correlations (CDX.NA.IG)**



Source: Merrill Lynch

**Chart 4: Base Correlations (iTraxx Europe)**



Source: Merrill Lynch

*Pricing non-standard tranches is more intuitive with base correlations*

**Base correlations provide a more intuitive way to estimate this correlation input and price the tranche.** Consider the pricing of a 5-8% tranche of the CDX.NA.IG3. A long position in a 5-8% tranche is essentially comprised of two first-loss tranches:

- long position in the 0-8%
- short position in the 0-5%

Each of these first-loss or base tranches needs a default correlation input to be priced. This input can be derived from the base correlations of the standardized tranches. Chart 3 and Chart 4 highlight that base correlations for the standardized tranches increase for increasing detachment points. As a result, the base

correlation of a 0-8% CDX tranche, for example, can be derived by interpolating on the curve between 7% and 10%. Similarly we can price the 0-5% tranche by interpolating on the curve between 3% and 7%. The 5-8% tranche can be priced in the following four steps:

- Interpolate between 0-7% and 0-10% to get the base correlation of the 0-8% tranche.
- Interpolate between 0-3% and 0-7% to get the base correlation of the 0-5% tranche.
- Use the standard Gaussian Copula model to price both tranches.
- The 5-8% tranche price is the difference in 0-8% and 0-5% prices adjusted for the notional of each tranche.

In order to price each of the base tranches, we need to rescale the notional of the base tranches (0-5% and 0-8%) in terms of the “target” tranche (5-8%) notional, yielding a width ratio of 1.67x and 2.67x for the two base tranches. Then, we compute the default and the risky DV01 of the 5-8% tranche as the difference of the legs of the two base tranches weighted by the corresponding width ratio. Finally, the spread of the 5-8% tranche is found by dividing the 5-8% default leg (5%) by its risky DV01 (4.22) which implies a breakeven spread of 110bps. Table 3 highlights this calculation in more detail.

We use the two **width ratios** to scale each intermediate step by the appropriate notional

Notional Scaled 5-8% DV01  
 $(3.64 \times 2.67) - (3.29 \times 1.67) = 4.22$

5-8% Breakeven Spread =  
 Default Leg / Risky DV01  
 $0.05 / 4.22 = 110 \text{ bps}$

**Table 3: Non-Standard 5-8% Tranche Breakeven Spread Calculation**

	Tranche #1	Tranche #2	Off-The-Run Tranche
Attachment Point	0%	0%	5%
Detachment Point	5%	8%	8%
Base Correlation	24.43%	31.2%	
Width Ratios	5% / 3%=1.67x	8% / 3%=2.67x	
Risky DV01	3.29	3.64	4.22
Default Leg	0.34	0.23	0.05
<b>Breakeven Spread</b>	1022 bps	625 bps	<b>110 bps</b>

Source: Assume a spread for the CDX.NA.IG 3 Index at 50.5 bps, 5y maturity and 40% average recovery.

In this analysis, we have assumed that the underlying portfolio is unchanged, i.e. the 125-name CDX.NA.IG. Portfolios with different underlying credits would make use of the correlation data from the observed standardized tranche market to generate a market-based estimate for the default correlation input. There are different ways in which this can be done. We leave this discussion for the future.

### Deltas: Base vs. Tranche Implied

By definition, pricing a tranche with either the tranche implied or base correlation should produce the same result. In fact, both measures are calibrated from the market in order to match traded prices.

This argument does not necessarily apply as far as sensitivity parameters are concerned. These refer to individual deltas for the underlying CDS or **leverage** with respect to the underlying index. Leverage is defined as the MTM of the tranche divided by the MTM of the underlying index for a 1bp change in the index.

As explained in an earlier publication<sup>2</sup>, the **tranche leverage** can also be derived using the following steps:

*Deltas depend on the chosen correlation measures*

<sup>2</sup> Please refer to “Correlation Trading” by Kakodkar/Martin/Galiani dated 26 November 2003 for an explanation of the concept and the computation of the tranche deltas and leverage.

1. Compute the **individual credit spread delta** for each underlying credit
2. Compute the tranche width as the difference of the tranche detachment and attachment points.
3. Compute the **average portfolio delta** and divide this number by the tranche width.

For the 3-7% levels in Table 1 and assuming a flat CDS term structure for all credits (equal to the CDX.NA.IG3 mid spread of 50.5bps), we find an average portfolio delta equal to 45.2%. The average delta divided by the tranche width (4%) implies a tranche leverage of about 11.3x.

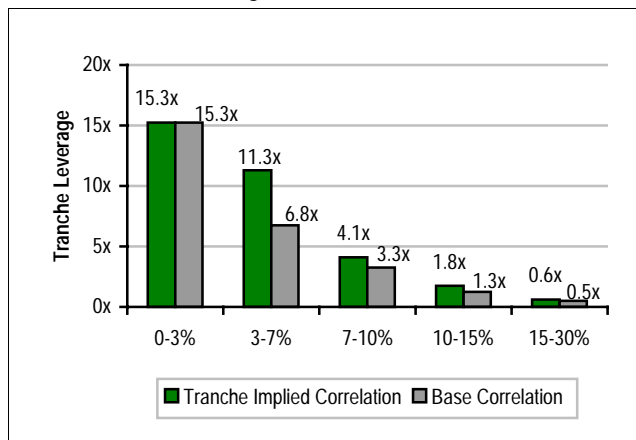
Within the base correlation framework, the computation of the 3-7% tranche leverage is slightly different and can be described as follows:

1. Compute the average of the individual credit spread deltas for the 0-7% tranche (68%) and for the 0-3% tranche (41%)
2. Compute the average individual delta for the 3-7% tranche (27%) as the difference of the average individual delta for the 0-7% and the 0-3% tranche
3. Divide the 3-7% average individual delta by the tranche width to get the leverage (6.75x).

*Under the base correlation framework tranche deltas tend to be lower*

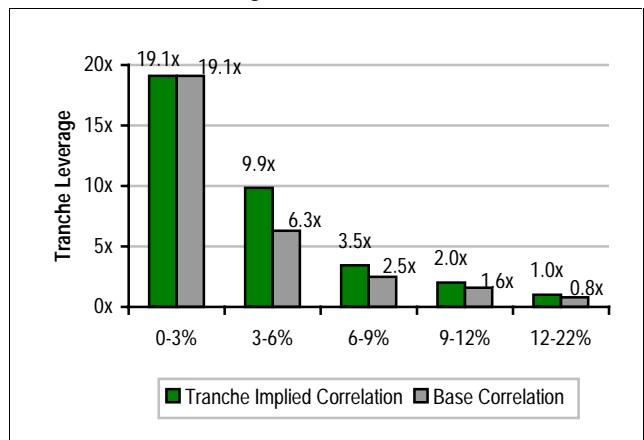
In Chart 5 and Chart 6, we compare the tranche leverage implied by using the two correlation measures for the North American and European market. The difference in leverage under the two methodologies (11.3x with the tranche implied correlation versus 6.8x for the base correlation) demonstrates the impact of each correlation measure in terms of risk management and hedging purposes. The charts below also highlight that the difference is most relevant for the mezzanine tranches (3-7% and 3-6%).

Chart 5: Tranche Leverage CDX.NA.IG3



Source: Merrill Lynch; Assume a flat CDS spread at 50.5bps for all credits.

Chart 6: Tranche Leverage iTraxx Series 2



Source: Merrill Lynch; Assume a flat CDS spread at 37bps for all credits.

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